57-34
p.2

N92-23343

Development of New Flux Splitting Schemes

Meng-S. Liou and Christopher J. Steffen, Jr.

1. Motivation and Objectives

Maximizing both accuracy and efficiency has been the primary objective in designing a numerical algorithm for computational fluid dynamics (CFD). This is especially important for solution of complex 3D systems of Navier-Stokes equations which often include turbulence modeling and chemistry effects. Recently, upwind schemes have been well received for both their capability of resolving discontinuities and their sound theoretical basis in characteristic theory for hyperbolic systems. With this in mind, we present two new flux splitting techniques for upwind differencing.

2. Work Accomplished

The first method is based upon High-Order Polynomial Expansions (HOPE) of the mass flux vector [1]. The present splitting results in positive and negative mass flux components that vanish at $M=0$. Thus the error in the Van Leer scheme which results in the diffusion of the boundary layer is eliminated. We also introduce several choices for splitting the pressure and examine their effects on the solution.

The second new flux splitting is based on the Advection Upwind Splitting Method (AUSM for short) [2]. In Navier-Stokes calculations, the diffusion error present in Van Leer's flux splitting scheme corrupts the velocity vector near the wall. In the AUSM, a proper splitting of the advective velocity component leads to an accurate resolution of the interface fluxes. The interface velocity is defined using the Mach number polynomial expansion in the mass flux, then the convective fluxes follow directly. Again, several choices of pressure splitting are possible among which a simple Mach number splitting according to characteristics appears to be the best in terms of accuracy. The scheme has yielded results whose accuracy rivals, and in some cases surpasses that of Roe's method, at reduced complexity and computational effort. The calculation of the hypersonic conical flow demonstrates the accuracy of the splittings in resolving the flow in the presence of strong gradients. The second series of tests involving the 2D inviscid flow over a NACA 0012 airfoil demonstrate the ability of the AUSM to resolve the shock discontinuity at transonic speed and the level of entropy generation at the stagnation point.

In the third case we calculate a series of supersonic flows over a circular cylinder. The Roe splitting in all conditions and grids tested yielded anomalous solutions (sometimes referred to as the carbuncle phenomenon), which could appear as non-symmetric, protuberant, or indented contours. The AUSM gave expected solutions in all calculations.

The fourth test deals with a 2D shock wave/boundary layer interaction. This provides an opportunity to accurately resolve a laminar separation region and to compare the ability to resolve a non grid-aligned shock with other methods.

3. Future Plans

Future plans are primarily concerned with the AUSM. A detailed stability analysis for this new technique will be useful. The idea of splitting the *advective velocity* opens up a whole family of potential schemes. Therefore, a comprehensive study of the interaction between various pressure and advection velocity splitting methods is necessary to optimize both accuracy and efficiency. Additionally, a 2D turbulent calculation would be a good test of the scheme's ability to solve a coupled system of $\kappa - \epsilon$ equations.

4. Publications

1. Liou, M.-S. and Steffen, C.J.Jr., "High-Order Polynomial Expansions (HOPE) for Flux-Vector Splitting," (to be presented at the ICES'91 Conference, August 11-16, 1991)
2. Liou, M.-S. and Steffen, C.J.Jr., "Development of a New Flux Splitting Scheme," (to be presented at the AIAA Tenth CFD Conference, June 24-26, 1991)